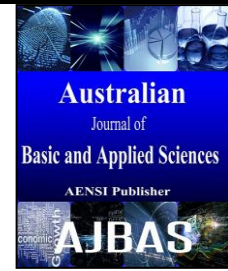




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### Effect of Mosquito Repellant on the Dynamics Model of Malaria

<sup>1</sup>Sunisa Inprasang, <sup>2</sup>Kadkanok Noodee and <sup>3</sup>Surapol Naowarat

<sup>1,2,3</sup> Department of Mathematics, Faculty of Science and Technology, Suratthani Rajabhat University, Suratthani, Thailand. 84100

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#### ABSTRACT

A nonlinear mathematical with mosquito repellant that describes the transmission of Malaria is proposed and analyzed. The human population is divided into three compartments depending on disease status, susceptible, infected and recovered. The mosquito population is divided into two compartments, susceptible and infected. We take into account the effect of mosquito repellant in our model. The stability of our model is determined by using the Routh–Hurwitz criteria. We applied the next generation method and the spectral radius, which give the basic reproductive number ( $\mathcal{R}_0$ ). The stability conditions both disease free equilibrium and endemic equilibrium are found and their stabilities are investigated, which depended on the basic reproductive number. If  $\mathcal{R}_0 < 1$ , the disease free equilibrium point is local asymptotically stable that mean, the disease will die out. But if  $\mathcal{R}_0 > 1$ , there exist the endemic equilibrium, which is local asymptotically stable that there will be Malaria outbreak. The numerical simulations are presented to illustrate the analytic results. In addition, we show that when the effectiveness of mosquito repellant is increase then the number infected human will be decrease.

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#### INTRODUCTION

Malaria is an infectious disease caused by the Plasmodium parasite and transmitted between humans through the bite of the female Anopheles mosquito. Symptoms of malaria typically include fever, fatigue, vomiting and headaches. In severe cases it can cause yellow skin, seizures, coma or death. These symptoms usually begin ten to fifteen days after being bitten (CDC, 2014). According to the latest estimates, released in December 2014, there were about 198 million cases of malaria in 2013 and an estimated 584,000 deaths. Malaria mortality rates have fallen by 47% globally since 2,000 and by 54% in the WHO African Region (WHO, 2014). Malaria is a major public health problem in Thailand. It has not been eradicated for many reasons. First, Thailand has the physical features of the land that is suitable for mosquitoes' to breed in. Parts of the population are also at a higher risk, for example the migrant worker and people who work in the forest. Finally, malaria is developing resistance to the malaria drugs. Malaria in Thailand is found along the border with Burma, Cambodia and Malaysia (Price *et al.*, 2007).

Mathematical models for transmission dynamics of malaria are useful in providing better insights into

the behavior of the disease. The models have played great roles in influencing the decision making processes regarding intervention strategies for preventing and controlling the insurgence of malaria (Olaniyi and Obabiyi, 2013). Many epidemic models have been analyzed mathematically and applied to specific diseases (Gupta *et al.*, 1994.; Hethcote, 2000). Since the first mathematical model of malaria transmission is introduced by Ross (1916). He introduced the first deterministic two-dimensional model with one variable representing humans and the other representing mosquitoes where it was shown that reduction of mosquito population below a certain threshold was sufficient to eradicate malaria. Ngwa and Shu (2000) analyze a deterministic differential equation model for endemic malaria involving variable human and mosquito populations. Oduro *et al.* (2012) proposed the mathematical model to investigate the transmission dynamics of malaria in Ghana is formulated taking into account human and mosquito populations. Stability analysis of the model is performed and make use of the next generation method to derive the basic reproduction number. Chiyaka *et al.* (2008) proposed the mathematical analysis of the effects of control strategies on the transmission dynamics of malaria. Model analysis

**Corresponding Author:** Kadkanok Noodee, Department of Mathematics, Faculty of Science and Technology, Suratthani Rajabhat University, Suratthani, 84100, Thailand.

E-mail: Kadkanok\_002@hotmail.com.

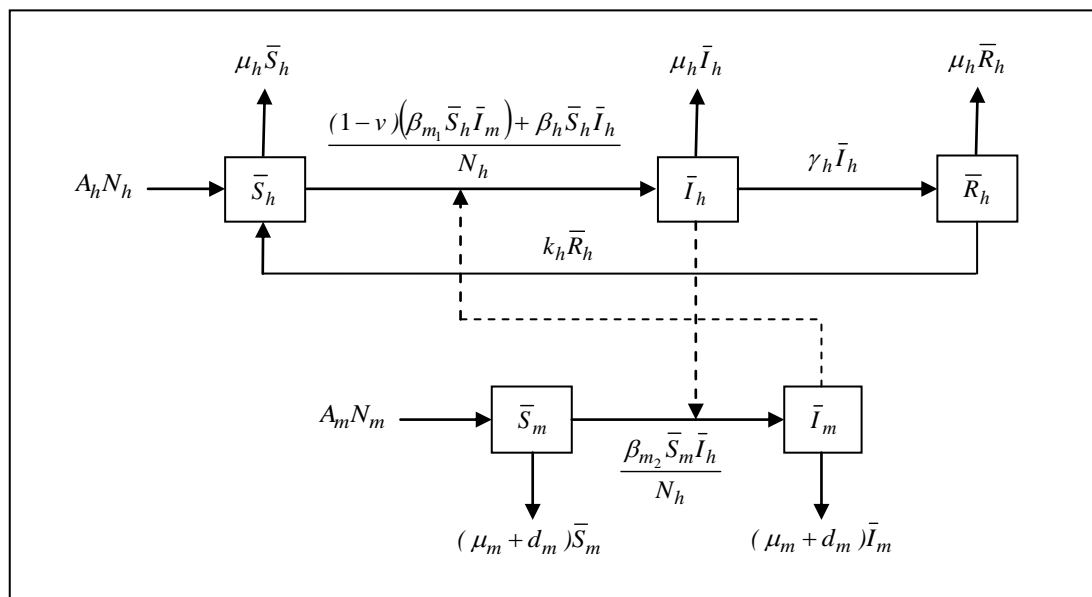
was qualitatively to determine criteria for control of a malaria epidemic and used to compute the threshold vaccination and treatment rates necessary for community-wide control of malaria. Tumwiine *et al* (2005) used an SIS and SI model in the human hosts and mosquito vectors respectively, for the study of malaria epidemic that lasts for a short period in which birth and immunity to the disease were ignored.

In this study, we are interested in the role of applying mosquito repellent on the dynamical of disease and how the efficacy of mosquito repellent relates to prevent and control the spread of disease. we introduce vital (birth and death) dynamics both in the human and mosquito populations. The infected humans either acquire some immunity or are susceptible again since immunity to malaria needs continuous exposure to reinfection. They may also die from the disease. Thus our model is based on the susceptible-infective-immune SIRS in human population and SI for the mosquito vector population. The recovery rate corresponds to how quickly parasites are cleared from the human host due to treatment.

The objective of the study is to determine the effect of mosquito repellent on the dynamics model of malaria. The remainder of the paper is organized as follows. In section 2, we formulate the propose model. In section 3, we analyze the model by using the standard method, to determine both disease free and endemic equilibrium point, derive the basic reproductive number and investigate the stability of the model. In section 4, we simulate the numerical results, which confirm our theoretical results. Finally, we conclude our study in section 5.

#### Model Formulaion:

We introduce a mathematical model of malaria. Because hosts might get repeatedly infected due to not acquiring complete immunity. For our model, we described by the SIRS model. Mosquitoes are assumed not to recover from the parasites so the mosquito population can be described by the SI model. The total number of population at time  $t$  is given by  $N_h = \bar{S}_h + \bar{I}_h + \bar{R}_h$  and  $N_m = \bar{S}_m + \bar{I}_m$ . The flow chart of model is shown in Fig 1.



**Fig. 1:** Flow chart for the transmission of Malaria

The human population is divided into the susceptible human ( $\bar{S}_h$ ), the infected human ( $\bar{I}_h$ ) and the recover human population ( $\bar{R}_h$ ) compartment.

The mosquito population is divided into two compartments, the susceptible mosquito ( $\bar{S}_m$ ) and the infected mosquito ( $\bar{I}_m$ ). The model does not

include immune as mosquitoes never recover from infection, that is, their infective period ends with their death due to their relatively short life-cycle.

The transmission dynamics of the Malaria are described by the following ordinary differential equations

$$\frac{d\bar{S}_h}{dt} = A_h N_h - \frac{(1-v)(\beta_{m_1} \bar{S}_h \bar{I}_m) + \beta_h \bar{S}_h \bar{I}_h}{N_h} - \mu_h \bar{S}_h + k_h \bar{R}_h \quad (1a)$$

$$\frac{d\bar{I}_h}{dt} = \frac{(1-v)(\beta_{m_1}\bar{S}_h\bar{I}_m) + \beta_h\bar{S}_h\bar{I}_h}{N_h} - \gamma_h\bar{I}_h - \mu_h\bar{I}_h \quad (1b)$$

$$\frac{d\bar{R}_h}{dt} = \gamma_h\bar{I}_h - \mu_h\bar{R}_h - k_h\bar{R}_h \quad (1c)$$

$$\frac{d\bar{S}_m}{dt} = A_m N_m - \frac{\beta_{m_2}\bar{S}_m\bar{I}_h}{N_h} - (\mu_m + d_m)\bar{S}_m \quad (1d)$$

$$\frac{d\bar{I}_m}{dt} = \frac{\beta_{m_2}\bar{S}_m\bar{I}_h}{N_H} - (\mu_m + d_m)\bar{I}_m \quad (1e)$$

The Eq.(1c) and (1d) can be canceled since the human and mosquito population are constant.i.e.,  $\bar{R}_h = N_h - \bar{S}_h - \bar{I}_h$  and  $\bar{S}_m = N_m - \bar{I}_m$ . By doing this we get three dependent variables, which we pick to be  $S_h$ ,  $I_h$ ,  $I_m$

To analyze the model by normalizing the Eq. (1a), (1b), (1e) and defining new variables

$$S_h = \frac{\bar{S}_h}{N_h}, I_h = \frac{\bar{I}_h}{N_h}, R_h = \frac{\bar{R}_h}{N_h}, S_m = \frac{\bar{S}_m}{N_m}, I_m = \frac{\bar{I}_m}{N_m}.$$

Since the total human and mosquito populations are constant, thus the time rate of change of human and mosquito population equal to zero, i.e.,  $\frac{d\bar{S}_h}{dt} + \frac{d\bar{I}_h}{dt} + \frac{d\bar{R}_h}{dt} = 0$  and  $\frac{d\bar{S}_m}{dt} + \frac{d\bar{I}_m}{dt} = 0$ . It means that the birth rate and the death rate of human and mosquito population are equal,  $A_h = \mu_h$  and  $A_m = (\mu_m + d_m)$ .

Our model has the following variables and parameters as follows:

$S_h(t)$  is the number of susceptible human at time  $t$ ,

$I_h(t)$  is the number of infected human at time  $t$ ,

$R_h(t)$  is the number of recovered (immune) human at time  $t$ ,

$S_m(t)$  is the number of susceptible mosquito at time  $t$ ,

$I_m(t)$  is the number of infected mosquito at time  $t$ ,

$N_h(t)$  is the total human population at time  $t$ ,

$N_m(t)$  is the total mosquito population at time  $t$ ,

$A_h$  is the birth rate of human population,

$\mu_h$  is the natural death rate of human population,

$A_m$  is the birth rate of mosquito population,

$\mu_m$  is the natural death rate of mosquito population,

$\gamma_h$  is the recovery rate of human population,

$k_h$  is the rate of loss immunity of human population,

$d_m$  is the death rate from spray of mosquitoes,

$v$  is the effective rate of use mosquito repellent for protecting in human population,

$m = \frac{N_m}{N_h}$  is the number of mosquitoes per human,

$\beta_h$  is the probability of transmission rate from an infectious human to a susceptible human,

$\beta_{m_1}$  is the probability of transmission rate from an infectious mosquito to a susceptible human,

$\beta_{m_2}$  is the probability of transmission rate from an infectious human to a susceptible mosquito.

The reduced model is depicted as follows:

$$\frac{dS_h}{dt} = A_h - (1-v)(m\beta_{m_1}S_hI_m) - \beta_hS_hI_h + k_h(1-S_h-I_h) - \mu_hS_h \quad (2)$$

$$\frac{dI_h}{dt} = (1-v)(m\beta_{m_1}S_hI_m) + \beta_hS_hI_h - \gamma_hI_h - \mu_hI_h \quad (3)$$

$$\frac{dI_m}{dt} = (1-I_m)\beta_{m_2}I_h - (\mu_m + d_m)I_m \quad (4)$$

with  $S_h + I_h + R_h = 1$  and  $S_m + I_m = 1$ .

### Model Analysis:



We obtained the basic reproduction number as show,  $\mathfrak{R}_0 = \sqrt{R_0}$

where

### 3.3 Local stability of equilibrium points:

#### 3.3.1 Local stability of disease free equilibrium:

The local stability of the disease free equilibrium can be analyzed using the Jacobian matrix of the malaria model at the disease free equilibrium point. Referring to the results of Van den Driessche and Watmough (2002), the following theorem holds.

#### Theorem 1:

$$J_0 = \begin{bmatrix} -\mu_h - k_h & -(\beta_h + k_h) & -(1-v)m\beta_{m_1} \\ 0 & -\gamma_h - \mu_h + \beta_h & (1-v)m\beta_{m_1} \\ 0 & \beta_{m_2} & -(\mu_m + d_m) \end{bmatrix}.$$

The eigenvalues of the  $J_0$  are obtained by solving  $\det(J_0 - \lambda I) = 0$ . We obtained the characteristic equation:

$$(\lambda + (\mu_h + k_h))(\lambda^2 + B_1\lambda + B_2) = 0 \quad (12)$$

where

$$B_1 = \gamma_h + \mu_h + \mu_m + d_m - \beta_h,$$

$$B_2 = (\gamma_h + \mu_h - \beta_h)(\mu_m + d_m) - (1-v)m\beta_{m_1}\beta_{m_2}.$$

From the characteristic Eq. (12), we get one of eigenvalue is  $\lambda_1 = -(\mu_h + k_h) < 0$ . Next, to determine the other eigenvalues from the characteristic equation  $\lambda^2 + B_1\lambda + B_2 = 0$ . The roots of this equation are negative if it is satisfied with two conditions of Routh-Hurwitz criteria (Allen, 2006). Thus,  $E_0$  is

$$J_1 = \begin{bmatrix} -(1-v)m\beta_{m_1}I_m^* - \beta_h I_h^* - \mu_h - k_h & -(\beta_h S_h^* + k_h) & -(1-v)m\beta_{m_1}S_h^* \\ (1-v)m\beta_{m_1}I_m^* + \beta_h I_h^* & -\gamma_h - \mu_h + \beta_h S_h^* & (1-v)m\beta_{m_1}S_h^* \\ 0 & (1-I_m^*)\beta_{m_2} & -\beta_{m_2}I_h^* - (\mu_m + d_m) \end{bmatrix}.$$

The characteristic equation of Jacobian matrix at  $E_1$ , is given by solving  $\det(J_1 - \lambda I) = 0$ , we obtained :

$$\lambda^3 + C_1\lambda^2 + C_2\lambda + C_3 = 0 \quad (13)$$

where

$$C_1 = c_1 + c_2 + c_3, \quad C_2 = c_2c_3 - c_5c_7 + (c_2 + c_3)c_1 - c_4c_6, \quad C_3 = c_5c_6c_7 + (c_2c_3 - c_5c_7)c_1 - c_3c_4c_6,$$

$$c_1 = (1-v)m\beta_{m_1}I_m^* + \beta_h I_h^* + \mu_h + k_h, \quad c_2 = \gamma_h + \mu_h - \beta_h S_h^*, \quad c_3 = \beta_{m_2}I_h^* + \mu_m + d_m,$$

$$c_4 = -\beta_h S_h^* - k_h, \quad c_5 = (1-v)m\beta_{m_1}S_h^*, \quad c_6 = (1-v)m\beta_{m_1}I_m^* + \beta_h I_h^*,$$

$$c_7 = (1-I_m^*)\beta_{m_2}, \quad C_1 = c_1 + c_2 + c_3.$$

The three eigenvalues of  $\lambda^3 + C_1\lambda^2 + C_2\lambda + C_3 = 0$  have negative real part if they satisfy the Routh-Hurwitz criteria. Thus,  $E_1$  is local asymptotically stable for  $\mathfrak{R}_0 > 1$  if  $\lambda^3 + C_1\lambda^2 + C_2\lambda + C_3 = 0$  satisfies the following conditions:

- 1)  $C_1 > 0$ ,
- 2)  $C_3 > 0$ ,

$$\mathfrak{R}_0 = \frac{(1-v)m\beta_{m_1}\beta_{m_2}}{(\mu_m + d_m)(\gamma_h + \mu_h - \beta_h)}. \quad (11)$$

The disease free equilibrium point for the malaria model is locally asymptotically stable if  $\mathfrak{R}_0 < 1$  and unstable if  $\mathfrak{R}_0 > 1$ .

Proof. The Jacobian matrix of the malaria model (Eq.2-4) evaluated at the disease-free equilibrium point  $E_0(1,0,0)$ , is obtained as

local asymptotically stable for  $\mathfrak{R}_0 < 1$  if

$\lambda^2 + B_1\lambda + B_2 = 0$  satisfies the following conditions:

- 1)  $B_1 > 0$ ,
- 2)  $B_2 > 0$ .

#### 3.3.2 Local stability of endemic equilibrium:

#### Theorem 2:

The endemic equilibrium point for the malaria model (Eq.2-4) is locally asymptotically stable if  $\mathfrak{R}_0 > 1$  and unstable if  $\mathfrak{R}_0 < 1$ .

Proof. The Jacobian matrix of the malaria model (Eq.2-4) evaluated at the endemic equilibrium point  $E_1(S_h^*, I_h^*, I_m^*)$ , is obtained as

- 3)  $C_1C_2 > C_3$ .

### 4. Numerical Results:

In this study, we are interested in the transmission of malaria with the effect of efficacy of mosquito repellent for protecting the mosquito. The parameters are used for simulation as shown in table 1.

**Table 1:** Parameter values is used in numerical simulations at Disease Free State.

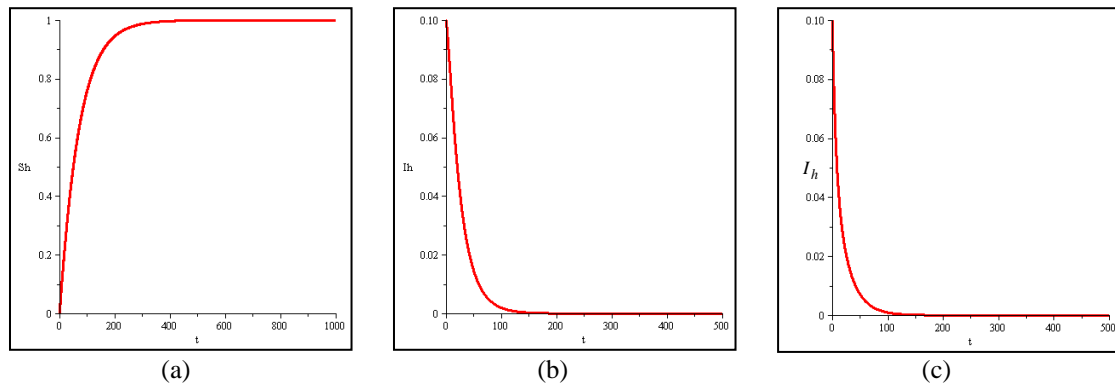
Parameter	Description	Values
$N_h(t)$	Total number of human population at time $t$	1,000
$N_m(t)$	Total number of mosquito population at time $t$	10,000
$A_h$	Birth rate of human population	$1/(65*365)$ day <sup>-1</sup>
$\mu_h$	Natural death rate of human population	$1/(65*365)$ day <sup>-1</sup>
$A_m$	Birth rate of mosquito population	1/7 day <sup>-1</sup>
$\mu_m$	Natural death rate of mosquito population	0.132857 day <sup>-1</sup>
$\gamma_h$	Recovery rate of human population	1/14 day <sup>-1</sup>
$k_h$	Rate of loss immunity for human population	0.0146 day <sup>-1</sup>
$d_m$	Death rate from spray for mosquito population	0.01 day <sup>-1</sup>
$v$	Effective rate of use mosquito repellent for protecting in human population	0.9
$\beta_h$	Probability of transmission rate from an infectious human	0.01
$\beta_{m_1}$	Probability of transmission rate from an infectious mosquito to a susceptible human	0.05
$\beta_{m_2}$	Probability of transmission rate from an infectious human to a susceptible mosquito	0.05

Stability of disease free state : From the values of parameters listed in Table 1, we obtained the eigenvalues and basic reproductive number is:

$$\lambda_1 = -0.014748 \quad \lambda_2 = -0.166630 \quad \lambda_3 = -0.037697, \mathfrak{R}_0 = 0.533562 < 1$$

Since all of these eigenvalues are to be negative and the basic reproductive number to be less than one, the equilibrium state will be the disease free state,  $E_0(1,0,0)$  as shown in Fig. 2 (a) Susceptible human ( $S_h$ ), (b) Infected human ( $I_h$ ) and Infected

mosquito ( $I_m$ ), proportion. The values of parameters are in the text. The solutions converge to the disease free equilibrium state as shown.

**Fig. 2:** Time series evolution of the population compartments of the case at the disease free state.

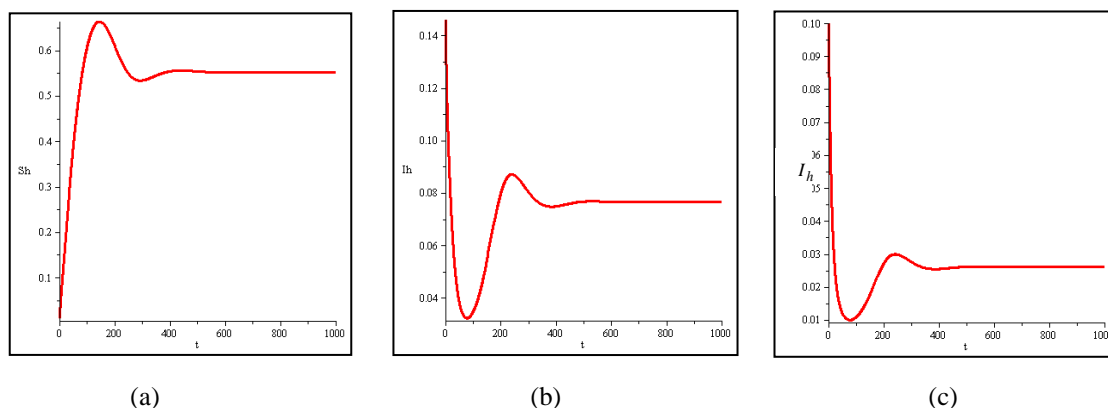
Stability of endemic state: Next we change the value of efficacy rate of the use of mosquito repellent for protecting in human population to be  $v = 0.3$ . The

other values of parameters are listed in Table 1, we obtained the eigenvalues and basic reproductive number is:

$$\lambda_1 = -0.213428 \quad \lambda_2 = -0.0119228 - 0.0214822i \quad \lambda_3 = -0.0119228 + 0.0214822i, \mathfrak{R}_0 = 1.41167 > 1.$$

Since all of these eigenvalues are to be negative and the basic reproductive number to be greater than one, the equilibrium state will be the endemic state,  $E_1$  as shown in Fig. 3. (a) Susceptible human, (b)

Infected human, (c) infected mosquito, proportion. The values of parameters are in the text, the solutions oscillate to the endemic equilibrium state.



**Fig. 3:** Time series evolution of the population compartments of the case at the endemic state.

### Conclusion:

We derived and analyzed a mathematical model for the transmission and spread of malaria with effect of mosquito repellent and analyzed the analytical results by using standard modeling method. The basic reproductive number is obtained through the use of spectral radius of the next generation matrix. The basic reproductive number is

$$\mathcal{R}_0 = \sqrt{\frac{(1-\nu)m\beta_{m_1}\beta_{m_2}}{(\mu_m + d_m)(\gamma_h + \mu_h - \beta_h)}}. \quad \text{The basic}$$

reproductive number is threshold condition for determining the stability of the equilibrium points of the model which are shown in Fig. 2 and 3. Our simulation results shown that  $\mathcal{R}_0$  will be decrease when the effect of mosquito repellent. We found the value of  $\mathcal{R}_0$  were 0.53356, 1.41167 when  $\nu = 0.9, 0.3$ , respectively. Simulations of the model clearly shows that when the numbers of human which used the mosquito repellents increasing, it is decreasing the numbers of the susceptible human to receive the malaria. In Thailand, as malaria fever has an epidemic. These alternative intervention could decreasing the numbers of infected human by this control measure to eradicate the malaria (Naowarat, 2012).

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